TISHK INERNATIONAL UNIVERSITY

DEPARTMENT OF MECHATRONICS ENGINEERING

QUESTION BANK

FLUID MECHANICS

CHAPTER I (PROPERTIES OF FLUIDS)

Group A (5 marks)

1. **Define density. Or Mass density or specific mass**

1.4.1 Mass Density

The *density* (also known as *mass density or specific mass*) of a liquid may be defined as *the mass per unit volume* $\left(\frac{m}{V}\right)$ *at a standard temperature and pressure*. It is usually denoted by ρ (rho). Its units are kg/m^3, *i.e.*,

$$\rho = \frac{m}{V} \qquad ...(1.1)$$

2. **Define specific weight. Or weight density.**

1.4.2 Weight Density

The weight density (also known as specific weight) is defined as the *weight per unit volume at the standard temperature and pressure*. It is usually denoted by *w*.

$$w = g \qquad ...(1.2)$$

For the purposes of all calculations, relating to Hydraulics and hydraulic machines, the specific weight of water is taken as follows:

In S.I. Units: $w = 9.81\ kN/m^3$ (or $9.81 \times 10^{-6}\ N/mm^3$)

In M.K.S. Units: $w = 1000\ kg_f/m^3$

3. **Define specific gravity.**

1.5. SPECIFIC GRAVITY

Specific gravity is the ratio of the specific weight of the liquid to the specific weight of a standard fluid. It is dimensionless and has no units. It is represented by *S*.

For liquids, the standard fluid is pure water at 4°C.

$$\therefore \qquad \text{Specific gravity} = \frac{\text{Specific weight of liquid}}{\text{Specific weight of pure water}} = \frac{w_{liquid}}{w_{water}}$$

4. Write the unit of Kinematic viscosity and dynamic viscosity.

Kinematic Viscosity :

Kinematic viscosity is defined as the *ratio between the dynamic viscosity and density of fluid.* It is denoted by ν (called nu).

$$\text{Mathematically,} \quad \nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \qquad \text{...(1.6)}$$

Units of kinematic viscosity:

In SI units: m^2/s

In M.K.S. units: $m^2/sec.$

In C.G.S. units the kinematic viscosity is also known as stoke (= $cm^2/sec.$)

One stoke = $10^{-4}\ m^2/s$

Note: *Centistoke* means $\frac{1}{100}$ stoke.

Dynamic Viscosity unit is Poise 1 poise = 1 / 10 **Ns/m^2**

5. Define Specific volume.

1.4.3 Specific volume

It is defined as *volume per unit mass of fluid*. It is denoted by *v*.

$$\text{Mathematically,} \qquad v = \frac{V}{m} = \frac{1}{\rho}$$

6. Define viscosity.

1.6. VISCOSITY

Viscosity may be defined as the *property of a fluid which determines its resistance to shearing stresses*. It is a measure of the internal fluid friction which causes resistance to flow. It is primarily *due to cohesion and molecular momentum exchange between fluid layers*, and as flow occurs, these effects appear as shearing stresses between the moving layers of fluid.

7. What do you mean by capillarity?

Cpailarity is a phenomenon by which a liquid *(depending upon its specific gravity*) rises into a thin glass tube above or below its general level.

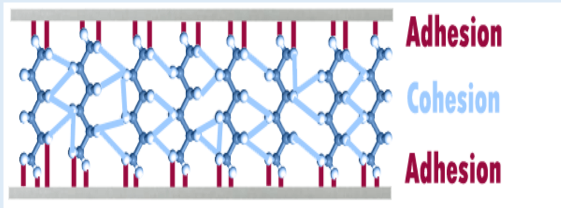
The capillarity happen due to *cohesion* and *adhesion* of liquid particles

8. What is surface tension?

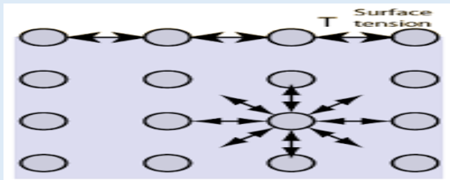
1.6 Surface Tension

Cohesion: intermolecular attraction between *molecules* of the same liquid.

Adhesion: attraction between the *molecules of a liquid and the Molecules* of *a solid boundary surface* In contact with the liquid



Surface tension: is caused by the *force of cohesion at the free surface*



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9. Calculate the work done in blowing a soap bubble of diameter 10cm.Assume the surface of soap solution=0.04 N/m.

Solution. *Given:* $d = 100$ mm or 0.1 m; $\sigma = 0.038$ N/m.

The soap bubble has two interfaces.

$\therefore$ Work done = Surface tension × total surface area

$$= 0.038 \times 4\pi \times \left(\frac{0.1}{2}\right)^2 \times 2$$

$$= \mathbf{0.002388\ Nm\ (Ans.)}$$

Group B

1. **Calculate the density,specific weight weight of one litre of petrol of specific gravity = 0.7**

Problem 1.21 *Calculate the density, specific weight and weight of one litre of petrol of specific gravity = 0.7*

Solution. Given : Volume = 1 litre = $1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity $S = 0.7$

(i) Density (ρ)

Using equation (1.1.A),

Density (ρ) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 =$ **700 kg/m³. Ans.**

(ii) Specific weight (w)

Using equation (1.1), $w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 =$ **6867 N/m³. Ans.**

(iii) Weight (W)

We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

or $w = \frac{W}{0.001}$ or $6867 = \frac{W}{0.001}$

∴ $W = 6867 \times 0.001 =$ **6.867 N. Ans.**

2. **A plate having an area of 0.6 m² is sliding down the inclined plate at 30⁰ to the horizontal with a velocity of 0.36 m/s. There is a cushion of fluid 1.8 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the plate is 280 N.**

Example 1.4. *A plate having an area of 0.6 m² is sliding down the inclined plane at 30° to the horizontal with a velocity of 0.36 m/s. There is a cushion of fluid 1.8 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the plate is 280 N.*

Solution: Area of plate, $A = 0.6 \text{ m}^2$

Weight of plate, $W = 280$ N

Velocity of plate, $u = 0.36$ m/s

Thickness of film, $t = dy = 1.8 \text{ mm} = 1.8 \times 10^{-3}$ m

Viscosity of the fluid, μ:

Component of W along the plate $= W \sin\theta = 280 \sin 30° = 140$ N

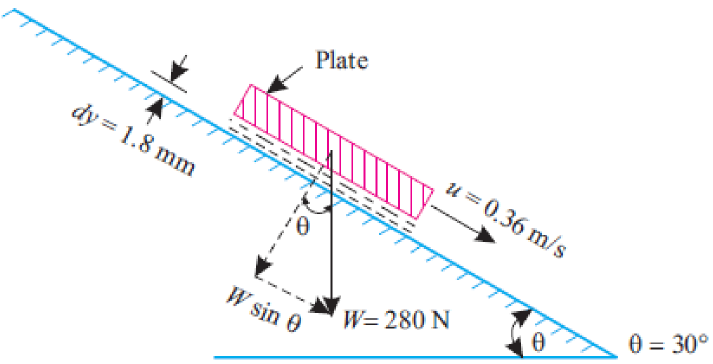


Fig. 1.4

∴ Shear force on the bottom surface of the plate, $F = 140$ N and shear stress,

$$\tau = \frac{F}{A} = \frac{140}{0.6} = 233.33 \text{N/m}^2$$

Where, du = change of velocity = $u - 0 = 0.36$ m/s

$dy = t = 1.8 \times 10^{-3}$ m

$$\therefore \quad 233.33 = \mu \times \frac{0.36}{1.8 \times 10^{-3}}$$

or, $$\mu = \frac{233.33 \times 1.8 \times 10^{-3}}{0.36} = 1.166 \text{ N.s/m}^2 = \mathbf{11.66 \text{ poise}}$$ **(Ans.)**

3. The space between two square flat parallel plates is filled with oil. Each side of the plate is 720 mm. The thickness of the oil film is 15 mm. The upper plate, which moves at 3 m/s requires a force of 120 N to maintain the speed. Determine

(i) The dynamic viscosity of the oil

(ii) The kinematic viscosity of oil if the specific gravity of oil is 0.95

Example 1.5. *The space between two square flat parallel plates is filled with oil. Each side of the plate is 720 mm. The thickness of the oil film is 15 mm. The upper plate, which moves at 3 m/s requires a force of 120 N to maintain the speed. Determine:*

(i) The dynamic viscosity of the oil;

(ii) The kinematic viscosity of oil if the specific gravity of oil is 0.95.

Solution. Each side of a square plate = 720 mm = 0.72 m

The thickness of the oil, dy = 15 mm = 0.015 m

Velocity of the upper plate = 3 m/s

$\therefore$ Change of velocity between plates, du = 3 – 0 = 3 m/s

Force required on upper plate, F = 120 N

$$\therefore \quad \text{Shear stress, } \tau = \frac{\text{force}}{\text{area}} = \frac{120}{0.72 \times 0.72} = 231.5 \text{ N/m}^2$$

(*i*) Dynamic viscosity, μ:

We know that,

$$\tau = \mu \cdot \frac{du}{dy}$$

$$231.5 = \mu \cdot \frac{3}{0.015}$$

$$\therefore \quad \mu = \frac{231.5 \times 0.015}{3} = \mathbf{1.16 \text{ N.s/m}^2}$$ **(Ans.)**

(*ii*) Kinematic viscosity, *v*:

Weight density of oil, $w = 0.95 \times 9.81 \text{ kN/m}^2 = 9.32 \text{ kN/m}^2$ = or 9320 N/m^3

$$\text{Mass density of oil, } \rho = \frac{w}{g} = \frac{9320}{9.81} = 950$$

Using the relation: $\quad \nu = \frac{\mu}{\rho} = \frac{1.16}{950} = 0.00122 \text{ m}^2/\text{s}$

Hence $\quad \nu =$ **0.00122 m^2/s (Ans.)**

4. **Two large fixed parallel planes are 12 mm apart. The space between the surfaces is filled with oil of viscosity 0.972 N.s/m^2. A flat thin plate 0.25 m^2 area moves through the oil at a velocity of 0.3 m/s. Calculate the drag force.**

(i)when the plate is equidistant from both the planes and

(ii)when the thin plate is at a distance of 4 mm from one of the plane surfaces.

Example 1.16. *Two large fixed parallel planes are 12 mm apart. The space between the surfaces is filled with oil of viscosity 0.972 N.s/m^2. A flat thin plate 0.25 m^2 area moves through the oil at a velocity of 0.3 m/s. Calculate the drag force:*

(*i*) *When the plate is equidistant from both the planes, and*

(*ii*) *When the thin plate is at a distance of 4 mm from one of the plane surfaces.*

Solution. Given: Distance between the fixed parallel planes = 12 mm = 0.012 m

Area of *thin* plate, $A = 0.25 \text{ m}^2$

Velocity of plate, $u = 0.3$ m/s

Viscosity of oil = 0.972 N.s/m^2

Drag force, *F*:

(*i*) *When the plate is equidistant from both the planes:*

Let, F_1 = Shear force on the upper side of the thin plate,

F_2 = Shear force on the lower side of the thin plate,

F = Total force required to drag the plate $(= F_1 + F_2)$.

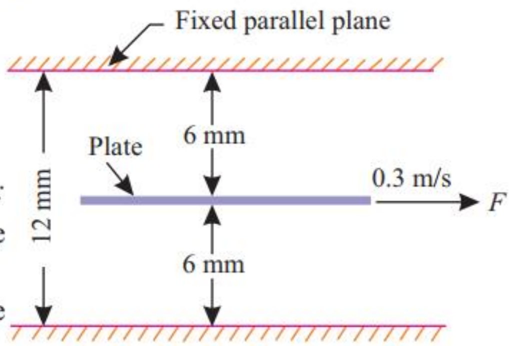


Fig. 1.12

5. A clean tube of diameter 2.5 mm is immersed in a liquid with a coefficient of surface tension = 0.4 N/m. The angle of the liquid with the glass can be assumed to be 135^0. The density of the liquid = 13600 kg/m^3. What would be the level of the liquid in the tube relative to the free surface of the liquid inside the tube.

Example 1.27. *A clean tube of diameter 2.5 mm is immersed in a liquid with a coefficient of surface tension = 0.4 N/m. The angle of contact of the liquid with the glass can be assumed to be 135°. The density of the liquid = 13600 kg/m^3.*

What would be the level of the liquid in the tube relative to the free surface of the liquid inside the tube.

Solution. *Given:* d = 2.5 mm ; σ = 4 N/m, θ = 135°; ρ = 13600 kg/m^3

Level of the liquid in the tube, *h*:

The liquid in the tube rises (or falls) due to capillarity. The capillary rise (or fall),

$$h = \frac{4\sigma\cos\theta}{wd} \quad \text{...[Eqn. (1.20)]}$$

$$= \frac{4 \times 0.4 \times \cos 135°}{(9.81 \times 13600) \times 2.5 \times 10^{-3}} \quad (\because w = \rho g)$$

$$= -3.39 \times 10^{-3} \text{m or } -3.39 \text{ mm}$$

Negative sign indicates that there is a capillary **depression (fall) of 3.39 mm. (Ans.)**

6. Calculate the specific weight, density and specific gravity of one litre of a liquid weighs 7 N

Problem 1.1 *Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7 N.*

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{m}^3 \quad \left(\because 1 \text{ litre} = \frac{1}{1000} \text{m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$$

$$\text{Weight} = 7 \text{ N}$$

(*i*) Specific weight (w) $= \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{7\text{ N}}{\left(\frac{1}{1000}\right)\text{m}^3}$ = **7000 N/m^3. Ans.**

(*ii*) Density (ρ) $= \dfrac{w}{g} = \dfrac{7000}{9.81}$ kg/m^3 = **713.5 kg/m^3. Ans.**

(*iii*) Specific gravity $= \dfrac{\text{Density of liquid}}{\text{Density of water}} = \dfrac{713.5}{1000}$ $\{\because$ Density of water = 1000 kg/m$^3\}$

= **0.7135. Ans.**

CHAPTER II (PRESSURE MEASUREMENTS)

Group A

1. **Define pascal's law.**

   It states that the pressure or intensity of pressure at a point in a static fluid is equal in all direction.

2. What are the types of pressure gauges?

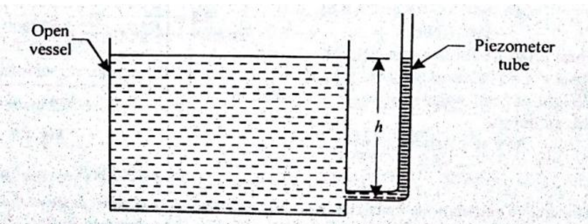
   1.Manaometrs

(*a*) Simple manometers:
(*i*) Piezometer, (*ii*) U-tube manometer, and (*iii*) Single column manometer.
(*b*) Differential manometers.

2.Mechanical gauges

(*i*) Bourdon tube pressure gauge, (*ii*) Diaphragm pressure gauge,
(*iii*) Bellow pressure gauge, and (*iv*) Dead-weight pressure gauge.

3. Draw the simple manometer.



(a)
Fig. 2.10. (*a*) Piezometer tube fitted to open vessel.

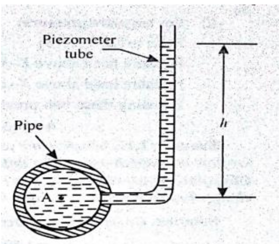
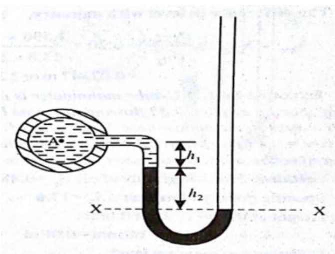


Fig. 2.10. (*b*) Piezometer tube fitted to a closed pipe.

4. Draw the U tube manometer for negative pressure.



$$h + h_1 S_1 + h_2 S_2 = 0 \quad \text{or} \quad h = -(h_1 S_1 + h_2 S_2)$$

5. What is differential manometer?

2.5.1.2. Differential Manometers

A differential manometer *is used to measure the difference in pressures between two points in a pipe, or in two different pipes.* In its simplest form a differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressures is required to be found out. Following are the most commonly used types of differential manometers.

1. U-tube differential manometer.
2. Inverted U-tube differential manometer.

Group B

1. **A U - tube manometer is used to measure the pressure of oil os specific gravity 0.85 flowing in a pipe line. Its left end is connected to the pipe and the right limb is open to the atmosphere. The centre of the pipe is 100 mm below the level of mercury(specific gravity = 13.6) in the right limb. If the difference of mercury level in the two limbs is 160 mm. Determine the absolute pressure of the oil in the pipe.**

Example 2.12. *A U-tube manometer is used to measure the pressure of oil of specific gravity 0.85 flowing in a pipe line. Its left end is connected to the pipe and the right-limb is open to the atmosphere. The centre of the pipe is 100 mm below the level of mercury (specific gravity = 13.6) in the right limb. If the difference of mercury level in the two limbs is 160 mm, determine the absolute pressure of the oil in the pipe.*

Solution. Specific gravity of oil, $S_1 = 0.85$

Specific gravity of mercury, $S_2 = 13.6$

Height of the oil in the left limb,

$$h_1 = 160 - 100 = 60 \text{ mm} = 0.06 \text{ m}$$

Difference of mercury level,

$$h_2 = 160 \text{ mm} = 0.16 \text{ m.}$$

Absolute pressure of oil:

Let, h = Gauge pressure in the pipe in terms of head of water, and

p = Gauge pressure in terms of kN/m^2.

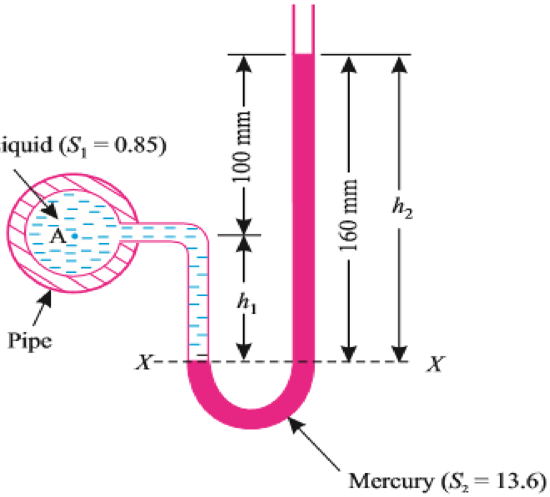


Fig. 2.13

Equating the pressure heads above the datum line X–X, we get:

$$h + h_1 S_1 = h_2 S_2$$

or, $h + 0.06 \times 0.85 = 0.16 \times 13.6 = 2.125$ m

The pressure p is given by:

$$p = wh$$
$$= 9.81 \times 2.125 \text{ kN/m}^2$$
$$= 20.84 \text{ kPa} \quad (\because w = 9.81 \text{ kN/m}^3 \text{ in S.I. units})$$

Absolute pressure of oil in the tube,

$$p_{abs.} = p_{atm.} + p_{gauge}$$

$$p_{abs.} = p_{atm.} + p_{gauge}$$

$$= 100 + 20.84 = \mathbf{120.84\ kPa}\ \textbf{(Ans.)}$$

2. U-tube manometer

containing mercury was used to find the negative pressure in the pipe, containing water. The right limb was open to the atmosphere. Find the vacuum pressure in the pipe, if the difference of mercury level in the two limbs was 100 mm and height of water in the left limb from the centre of the pipe was found to be 40 mm below.

Example 2.13. *U-tube manometer containing mercury was used to find the negative pressure in the pipe, containing water. The right limb was open to the atmosphere. Find the vacuum pressure in the pipe, if the difference of mercury level in the two limbs was 100 mm and height of water in the left limb from the centre of the pipe was found to be 40 mm below.*

Solution. Specific gravity of water, $S_1 = 1$

Specific gravity of mercury, $S_2 = 13.6$

Height of water in the left limb,

$h_1 = 40 \text{ mm} = 0.04 \text{ m}$

Height of mercury in the left limb,

$h_2 = 100 \text{ mm} = 0.1 \text{ m}$

Let, h = Pressure in the pipe in terms of head of water (*below* the atmosphere).

Equating the pressure heads above the datum line X–X, we get:

$$h + h_1S_1 + h_2S_2 = 0$$

or,

$$h = -(h_1S_1 + h_2S_2)$$
$$= -(0.04 \times 1 + 0.1 \times 13.6)$$
$$= -1.4 \text{ m of water}$$

Pressure p is given by:

$$p = wh$$
$$= 9.81 \times (-1.4) \text{ kN/m}^2$$
$$= -13.73 \text{ kPa}$$

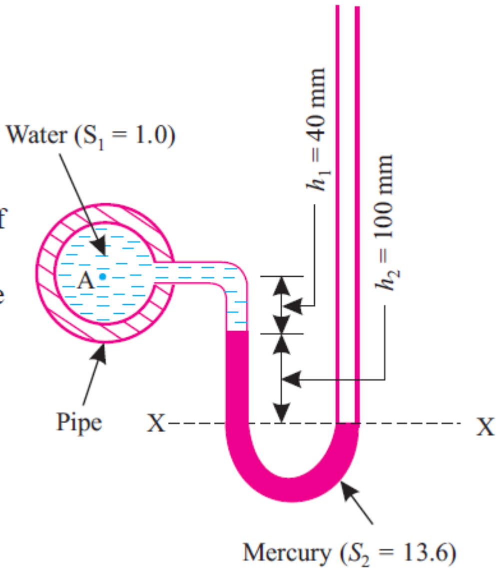


Fig. 2.14

3. Explain with neat sketch of different types of pressure gauges.

2.5 Measurement of Pressure

The pressure of a fluid may be measured by the following devices:

1. Manometers:

Manometers *are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of liquid.* These are classified as follows:

(*a*) Simple manometers:

(*i*) Piezometer, (*ii*) U-tube manometer, and (*iii*) Single column manometer.

(*b*) Differential manometers.

2. Mechanical gauges:

These are the devices in which the pressure is measured by balancing the fluid column by spring (elastic element) or dead weight. Generally these gauges are used for measuring *high pressure* and where *high precision is not required.* Some commonly used mechanical gauges are:

(*i*) Bourdon tube pressure gauge, (*ii*) Diaphragm pressure gauge,

(*iii*) Bellow pressure gauge, and (*iv*) Dead-weight pressure gauge.

Simple Manometer

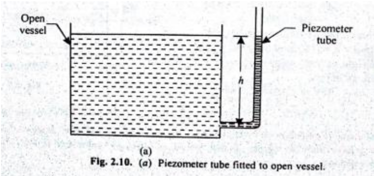


Fig. 2.10. (*a*) Piezometer tube fitted to open vessel.

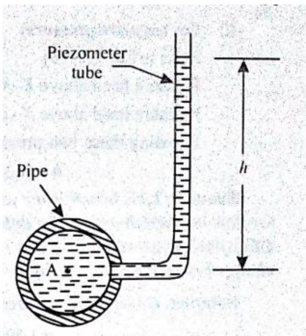
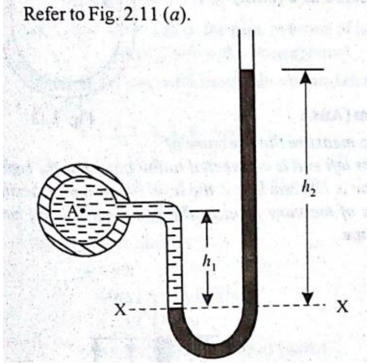


Fig. 2.10. (*b*) Piezometer tube fitted to a closed pipe.

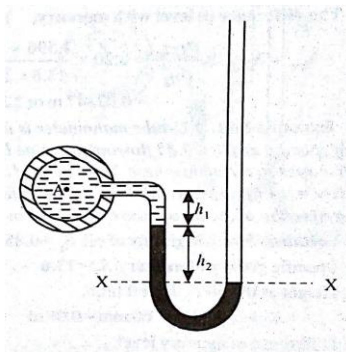
U Tube manometer

(*i*) **For positive pressure:**
Refer to Fig. 2.11 (*a*).



$$h + h_1 S_1 = h_2 S_2 \quad \text{or} \quad h = h_2 S_2 - h_1 S_1$$

For Negative pressure



$$h + h_1 S_1 + h_2 S_2 = 0 \quad \text{or} \quad h = -(h_1 S_1 + h_2 S_2)$$

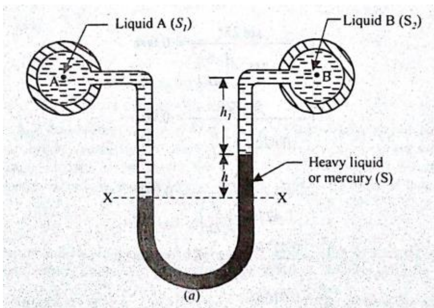
Differential Manometer

2.5.1.2. Differential Manometers

A differential manometer *is used to measure the difference in pressures between two points in a pipe, or in two different pipes.* In its simplest form a differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressures is required to be found out. Following are the most commonly used types of differential manometers.

1. U-tube differential manometer.
2. Inverted U-tube differential manometer.

U tube differential manometer

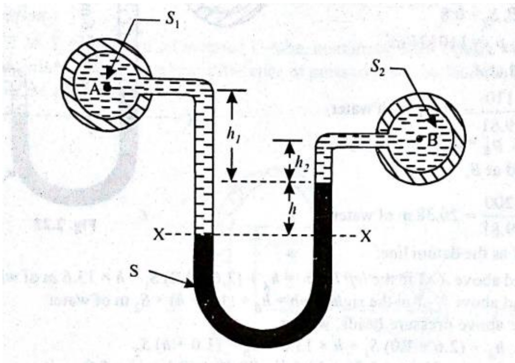


(*a*)

(*a*) Two pipes at same level.

Fig. 2.21. U-tube differential manometers.

$$h_A - h_B = h\,(S - S_1)$$



(*b*)

Fig. 2.21. (*b*) Two pipes at differential levels.

$$h_A - h_B = h\,(S - S_1) + h_2 S_2 - h_1 S_1$$

4. Figure shows a differential manometer connected at two points A and B. At A air pressure is 100 kN/m^2. Find the absolute pressure at B.

Example 2.21. *Fig. 2.23. shows a differential manometer connected at two points A and B. At A air pressure is 100 kN/m^2. Find the absolute pressure at B.*

Solution. Pressure of air at A,

$$p_A = 100 \text{ kN/m}^2$$

Pressure head at A,

$$h_A = \frac{100}{9.81} = 10.2 \text{ m}$$

Let the pressure at B is p_B.

Then, pressure head at $B = \frac{p_B}{w}$

Considering pressure heads above the atum line X–X, we have:

Pressure head in the *left limb*

$$= \frac{650}{1000} + h_A = 0.65 + 10.2 = 10.85 \text{ m}$$

Pressure head in the *right limb*

$$= h_B + \frac{250}{1000} \times 0.85 + \frac{150}{1000} \times 13.6$$

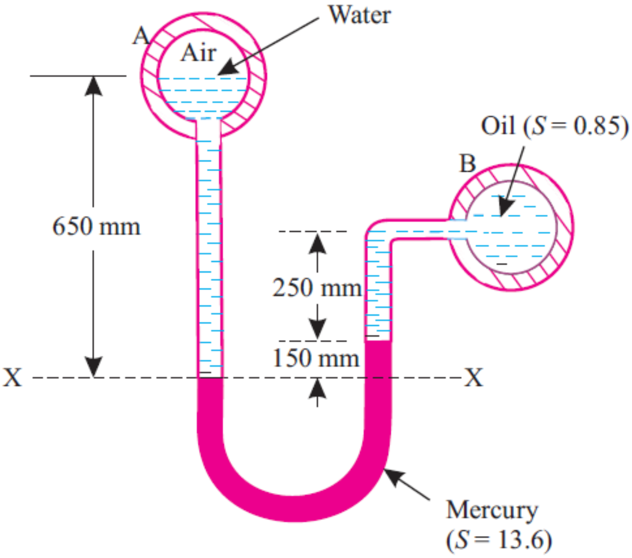


Fig. 2.23

$$= h_B + 0.212 + 2.04 = h_B + 2.25$$

Equating the above pressure heads, we get:

$$10.85 = h_B + 2.25 \quad \text{or} \quad h_B = 8.6 \text{ m}$$

But, $$h_B = \frac{P_B}{w}$$

$$p_B = wh_B = 9.81 \times 8.6 = 84.36 \text{ kN/m}^2$$

or, $$p_B = \mathbf{84.36 \text{ kPa}} \text{ (Ans.)}$$

Chapter III (Hydrostatic Forces on Surfaces)

Group A

1. Define centre of pressure.

Centre of pressure. It is defined as the *point of application of the total pressure on the surface.*

2. What do you mean by total pressure?

3.2 Total Pressure and Centre of Pressure

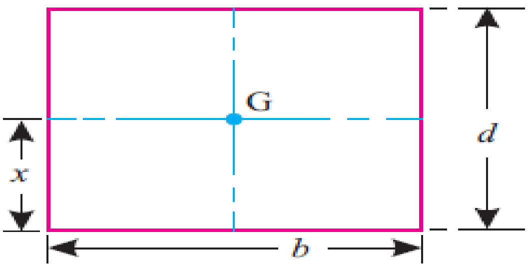
Total pressure. It is defined as the *force exerted by static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always at right angle (or normal) to the surface.*

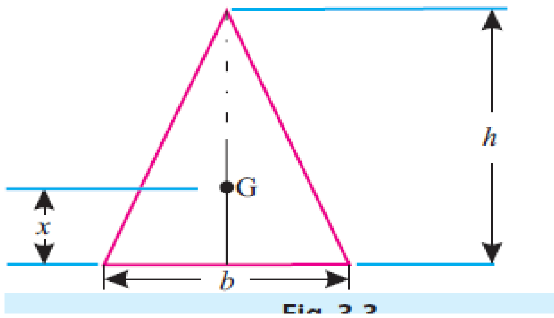
3. Name the different experimental setup of Centre of pressure.

Hydrostatic forces (Partially submerged)

Hydrostatic forces (Fully submerged)

4. Write the moment of inertia formula for rectangle and triangle about base.





S.No.	Name of figure	C.G. from the base	Area	I about an axis passing through C.G. and parallel to the base	I about base
1.	Triangle Fig. 3.3	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
2.	Rectangle Fig. 3.4	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$

Group B

1) A rectangular plate 3 m long and 1 m wide is immersed vertically in water in such a way that its 3 m side is parallel to the water surfaces and is 1 m below it. Find (i) Total pressure on the plate and (ii) Position of centre of pressure.

Example 3.2. *A rectangular plate 3 metres long and 1 metre wide is immersed vertically in water in such a way that its 3 metres side is parallel to the water surface and is 1 metre below it. Find: (i) Total pressure on the plate, and (ii) Position of centre of pressure.*

Solution. Width of the plane surface, $b = 3$ m

Depth of the plane surface, $d = 1$ m

Area of the plane surface,

$$A = b \times d = 3 \times 1 = 3 \text{ m}^2$$

$$\bar{x} = 1 + \frac{1}{2} = 1.5 \text{ m}$$

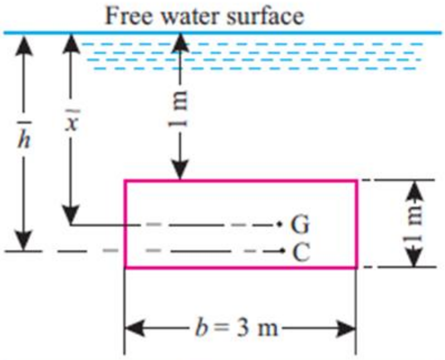


Fig. 3.8

(i) Total pressure P:

Using the relation:

$$P = wA\bar{x} = 9.81 \times 3 \times 1.5$$

$$= \mathbf{44.14 \text{ kN}} \text{ (Ans.)}$$

(ii) Centre of pressure, $\bar{h}$:

Using the relation: $$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

But, $$I_G = \frac{bd^3}{12} = \frac{3 \times 1^3}{12} = 0.25 \text{m}^4$$

$\therefore$ $$\bar{h} = \frac{0.25}{3 \times 1.5} + 1.5 = 1.556 \text{ m}$$

i.e. $$\bar{h} = \mathbf{1.556 \text{ m}} \text{ (Ans.)}$$

2) A trapezoidal 2 m wide at the bottom and 1 m deep has side slopes 1:1. Determine

(i)total pressure (ii)Centre of pressure on the vertical gate closing the channel when it is full of water.

Example 3.7. *A trapezoidal 2 m wide at the bottom and 1 m deep has side slopes 1 : 1. Determine:*
(i) Total pressure,
(ii) Centre of pressure on the vertical gate closing the channel when it is full of water.

Solution. Refer to Fig. 3.14

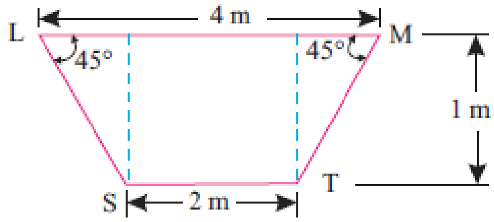


Fig. 3.14

(i) Total Pressure, P :

For rectangle:

Area, $A_1 = 2 \times 1 = 2\text{m}^2$

$\bar{x} = \frac{1}{2} = 0.5\text{m}$

$P_1 = wA\bar{x} = 9.81 \times 2 \times 0.5 = 9.81 \text{ kN}$

This acts at a depth $\bar{h}_1$.

But, $\bar{h}_1 = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(2 \times 1^3/12)}{2 \times 0.5} + 0.5 = 0.6 \text{ m}$ ... from the top

For triangles:

Area, $A_2 = 2 \times \frac{1}{2} \times 1 \times 1 = 1 \text{ m}^2$ (there are two triangles); $\bar{x} = \frac{1}{3}$ m

$P_2 = wA\bar{x} = 9.81 \times 1 \times \frac{1}{3} = 3.27 \text{ kN}$

This acts at a depth of $\bar{h}_2$.

But, $\bar{h}_2 = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(2 \times 1^3/36)}{1 \times 1/3} + \frac{1}{3} = 0.5 \text{ m}$ ...from the top.

i.e. $\bar{h}_2 = 0.5$ m

Total pressure, $P = P_1 + P_2 = 9.81 + 3.27 = \mathbf{13.08 \text{ kN}}$ **(Ans.)**

(ii) Centre of pressure, $\bar{h}$:

Taking moments about the top, we get: $P \times \bar{h} = P_1 \times \bar{h}_1 + P_2 \times \bar{h}_2$

or, $\bar{h} = \frac{P_1\bar{h}_1 + P_2\bar{h}_2}{P} = \frac{9.81 \times 0.66 + 3.27 \times 0.5}{13.08} = \mathbf{0.62 \text{ m}}$ **(Ans.)**

3) A triangle plate of 1 m base 1.5 m altitude is immersed in water. The plane of the plate is inclined at 30^0 with free water surface and the base is parallel to and at a depth of 2 m from water surface. Find the total pressure on the plate and the position of centre pressure.

Solution. Refer to Fig. 3.31.

Area of the plate,

$$A = \frac{1}{2} \times 1 \times 1.5 = 0.75 \text{ m}^2$$

Inclination of the plate, $\theta = 30°$

Total pressure on the plate, P:

The depth of *c.g.* of the plate from water surface,

$$\bar{x} = 2 + \frac{1.5}{3} \sin 30°$$

$$= 2 + 0.5 \times 0.5 = 2.25 \text{ m}$$

Using the relation,

$$P = wA\bar{x} = 9.81 \times 0.75 \times 2.25$$

$$= \mathbf{16.55 \text{ kN}} \text{ (Ans.)}$$

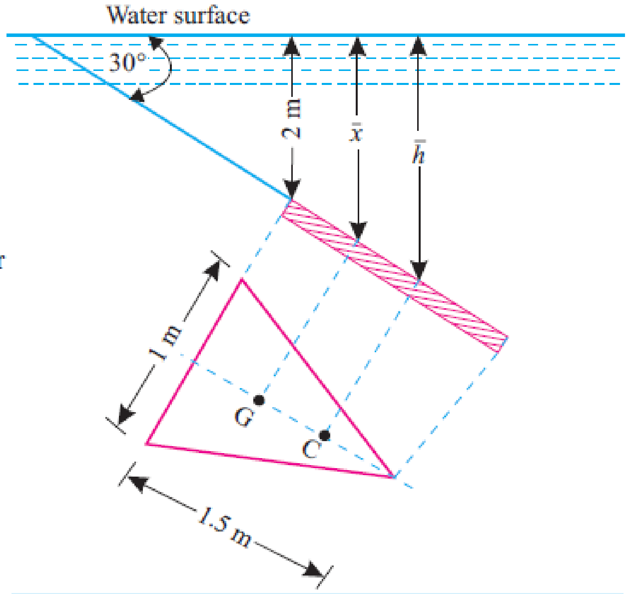


Fig. 3.31

Depth of centre of pressure, $\bar{h}$:

Moment of inertia of a triangular section about its *c.g.*,

$$I_G = \frac{1 \times 1.5^3}{36} = 0.09375 \text{ m}^4$$

Using the relation,

$$\bar{h} = \frac{I_G \sin^2 \theta}{A\bar{x}} + \bar{x} = \frac{0.09375 \sin^2 30°}{0.75 \times 2.25} + 2.25$$

$$= \mathbf{2.264 \text{ m}} \text{ (Ans.)}$$

4) Figure shows a circular plate of diameter 1.2 m placed vertically in water in such a way that the centre of the place is 2.5 m below the free surface of water. Determine

(i) Pressure on the plate (ii) position of centre of pressure.

Example 3.1. *Fig. 3.7 shows a circular plate of diameter 1.2 m placed vertically in water in such a way that the centre of the place is 2.5 m below the free surface of water. Determine: (i) Total pressure on the plate. (ii) Position of centre of pressure.*

Solution. Diameter of the plate, $d = 1.2$ m

Area,

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 1.2^2 = 1.13 \text{ m}^2$$

$$\bar{x} = 2.5 \text{ m}$$

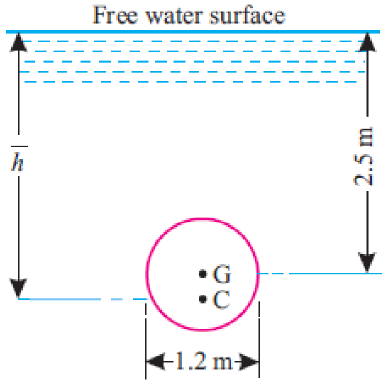


Fig. 3.7

(*i*) Total pressure, P:

Using the relation:

$$P = wA\bar{x} = 9.81 \times 1.13 \times 2.5$$

$$= \mathbf{27.7 \text{ kN}} \text{ (Ans.)}$$

(*ii*) Position of centre of pressure, $\bar{h}$:

Using the relation:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

where,

$$I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times 1.2^4 = 0.1018 \text{ m}^4$$

$$\bar{h} = \frac{0.1018}{1.13 \times 2.5} + 2.5 = 2.536 \text{ m}$$

i.e.

$$\bar{h} = \mathbf{2.536 \text{ m}} \text{ (Ans.)}$$

Chapter IV (FLUID KINEMATICS AND DYNAMICS)

Group A

1. What is the deference between laminar and turbulent flow.

5.3.5. Laminar and Turbulent Flows

Laminar flow. A laminar flow is one in which *paths taken by the individual particles do not cross one another and move along well defined paths* (Fig. 5.5), This type of flow is also called *stream-line flow or viscous flow*.

Examples. (*i*) *Flow through a capillary tube.*
(*ii*) *Flow of blood in veins and arteries.*
(*iii*) *Ground water flow.*

Turbulent flow. A turbulent flow is that flow in which fluid *particles move in a zig zag way* (Fig. 5.6).

Example. *High velocity flow in a conduit of large size. Nearly all fluid flow problems encountered in engineering practice have a turbulent character.*

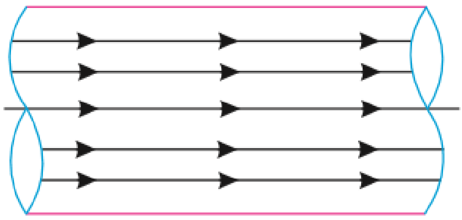


Fig. 5.5. Laminar flow.

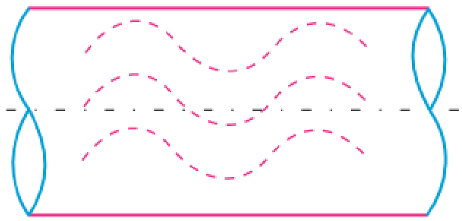


Fig. 5.6. Turbulent flow.

Laminar and turbulent flows are characterised on the basis of Reynolds number (refer to chapter 10).

For Reynolds number (Re) < 2000 ... flow in pipes is *laminar*.
For Reynolds number (Re) > 4000 ... flow in pipes is *turbulent*
For Re between 2000 and 4000 ... flow in pipes *may be laminar or turbulent*.

2. State the continuity equations.

The **continuity equation** *is based on the principle of conservation of mass.* It states as follows:

"If no fluid is added or removed from the pipe in any length then the mass passing across different sections shall be same."

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad ...(5.22)$$

Eqn. (5.22) is applicable to the compressible as well as incompressible fluids and is called ***Continuity Equation.*** In case of *incompressible fluids*, $\rho_1 = \rho_2$ and the continuity eqn. (5.21) reduces to:

$$A_1 V_1 = A_2 V_2 \quad ...(5.23)$$

3. Write the assumptions of Bernoullis Theorem.

Assumptions:

It may be mentioned that the following *assumptions* are made in the derivation of Bernoulli's equation:

1. The liquid is ideal and incompressible.
2. The flow is steady and continuous.
3. The flow is along the streamline, *i.e.*, it is one-dimensional.
4. The velocity is uniform over the section and is equal to the mean velocity.
5. The only forces acting on the fluid are the *gravity forces* and the *pressure forces*.

4. What is a venturimeters? Where it is used?

A venturimeter is one of the most important practical applications of Bernoulli's theorem. *It is an instrument used to measure the rate of discharge in a pipeline and is often fixed permanently at different sections of the pipeline to know the discharges there.*

5. What are the types of fluid flow?

TYPES OF FLUID FLOW

Fluids may be *classified* as follows:

1. Steady and unsteady flows
2. Uniform and non-uniform flows
3. One, two and three dimensional flows
4. Rotational and irrotational flows
5. Laminar and turbulent flows
6. Compressible and incompressible flows.

6. State Bernnoullis Theorem.

Bernoulli's equation states as follow:

"In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line."

Mathematically,

$$\frac{p}{w} + \frac{V^2}{2g} + z = \text{constant}$$

7.Write the expression of bernoullis equations.

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

Group B

1. Derive the expression for Bernoullis equation.

Proof.

Consider an ideal incompressible liquid through a non-uniform pipe as shown in Fig 6.1. Let us consider two sections LL and MM and assume that the pipe is running full and there is continuity of flow between the two sections;

Let,

p_1 = Pressure at LL,

V_1 = Velocity of liquid at LL,

z_1 = Height of LL above the datum,

A_1 = Area of pipe at LL, and

p_2, V_2, z_2, A_2 = Corresponding values at MM.

Let, W = Weight of liquid between LL and $L'L'$.

As the flow is continuous,

$$\therefore \quad W = wA_1 \cdot dl_1 = wA_2 \cdot dl_2$$

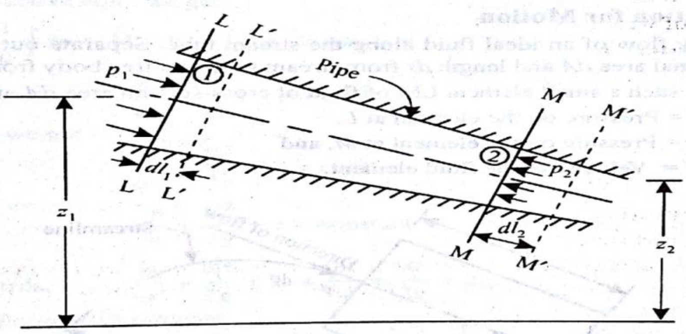
$$\text{or} \quad A_1 \cdot dl_1 = \frac{W}{w} \quad \text{...(i)}$$

$$\text{Similarly,} \quad A_2 \cdot dl_2 = \frac{W}{w} \quad \text{...(ii)}$$

$$\therefore \quad A_1 \cdot dl_1 = A_2 . dl_2$$

Work done by pressure at LL, in moving the liquid to $L'L'$

$$= \text{Force} \times \text{distance} = p_1 \cdot A_1 \cdot dl_1$$



Similarly, work done by the pressure at MM in moving the liquid to $M'M' = -p_2 . A_2 . dl_2$
(– ve sign indicates that direction of p_2 is *opposite* to that of p_1)

$\therefore$ Total work done by the pressure

$$= p_1 . A_1 \, dl_1 - p_2 A_2 \, dl_2$$
$$= p_1 . A_1 \, dl_1 - p_2 A_1 \, dl_1 \qquad (\because A_1 dl_1 = A_2 dl_2)$$
$$= A_1 . dl_1 (p_1 - p_2)$$
$$= \frac{W}{w}(p_1 - p_2) \qquad \left(\because A_1 . dl_1 = \frac{W}{w}\right)$$

Loss of potential energy $= W(z_1 - z_2)$

Gain in kinetic energy $= W\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) = \frac{W}{2g}(V_2^2 - V_1^2)$

Also, loss of potential energy + work done by pressure = gain in kinetic energy

$$\therefore \quad W(z_1 - z_2) + \frac{W}{w}(p_1 - p_2) = \frac{W}{2g}(V_2^2 - V_1^2)$$

or
$$(z_1 - z_2) + \left(\frac{p_1}{w} - \frac{p_2}{w}\right) = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right)$$

or
$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 \quad \text{...(6.2)}$$

which proves *Bernoulli's equation.*

2. Write short notes on different types of Fluid flow with examples and neat sketches.

Steady flow

the fluid characteristics like velocity, pressure, density, etc. at a point *do not change* with time is called *steady flow*. Mathematically, we have:

$$\left(\frac{\partial u}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial w}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0; \text{ and so on}$$

where (x_0, y_0, z_0) is a fixed point in a fluid field where these variables are being measured *w.r.t.* time.

Example. *Flow through a prismatic or non-prismatic conduit at a constant flow rate Q* m^3/s *is steady.*

(A prismatic conduit has a constant size shape and has a velocity equation in the form $u = ax^2 + bx + c$, which is independent of time t).

Unsteady flow. It is that type of flow in which the velocity, pressure or density at a point *change* w.r.t. time. Mathematically, we have:

$$\left(\frac{\partial u}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial w}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \text{ and so on}$$

Example. *The flow in a pipe whose valve is being opened or closed gradually* (*velocity equation is in the form* $u = ax^2 + bxt$).

5.3.2. Uniform and Non-uniform Flows

Uniform flow. The type of flow, in which the velocity at any given time *does not change* with respect to space is called *uniform flow*. Mathematically, we have:

$$\left(\frac{\partial V}{\partial s}\right)_{t\,=\,\text{constant}} = 0$$

where, ∂V = Change in velocity, and

∂s = Displacement in any direction.

Example. *Flow through a straight prismatic conduit (i.e. flow through a straight pipe of constant diameter).*

Non-uniform flow. It is that type of flow in which the velocity at any given time *changes* with respect to space. Mathematically,

$$\left(\frac{\partial V}{\partial s}\right)_{t\,=\,\text{constant}} \neq 0$$

Example. (*i*) *Flow through a non-prismatic conduit.*

(*ii*) *Flow around a uniform diameter pipe-bend or a canal bend.*

5.3.5. Laminar and Turbulent Flows

Laminar flow. A laminar flow is one in which *paths taken by the individual particles do not cross one another and move along well defined paths* (Fig. 5.5), This type of flow is also called *stream-line flow or viscous flow*.

Examples. (*i*) *Flow through a capillary tube.*

(*ii*) *Flow of blood in veins and arteries.*

(*iii*) *Ground water flow.*

Turbulent flow. A turbulent flow is that flow in which fluid *particles move in a zig zag way* (Fig. 5.6).

Example. *High velocity flow in a conduit of large size. Nearly all fluid flow problems encountered in engineering practice have a turbulent character.*

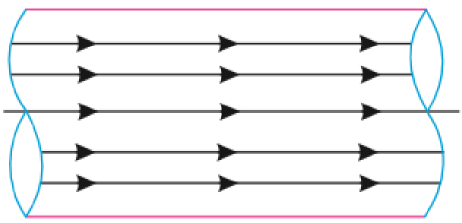


Fig. 5.5. Laminar flow.

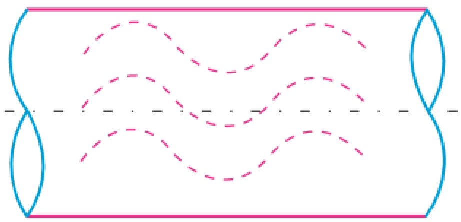


Fig. 5.6. Turbulent flow.

Laminar and turbulent flows are characterised on the basis of Reynolds number (refer to chapter 10).

For Reynolds number (Re) < 2000 ... flow in pipes is *laminar*.

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For Re between 2000 and 4000 ... flow in pipes *may be laminar or turbulent*.

3. Water flows in a circular pipe. At one section the diameter is 0.3 m the static pressure is 260 kpa gauge, the velocity is 3 m/s and the elevation is 10 m above ground level. The elevation at a section downstream is 0 m and the pipe diameter is 0.15 m. Find the gauge pressure at the downstream section. Frictional effect may be neglected. Assume density of water to be 999 kg/m^3.

Example 6.6. *Water flows in a circular pipe. At one section the diameter is 0.3 m, the static pressure is 260 kPa gauge, the velocity is 3 m/s and the elevation is 10 m above ground level. The elevation at a section downstream is 0 m, and the pipe diameter is 0.15 m. Find out the gauge pressure at the downstream section.*

Frictional effects may be neglected. Assume density of water to be 999 kg/m^3.

(RGPV, Bhopal)

Solution. Refer to Fig. 6.7. $D_1 = 0.3$ m; $D_2 = 0.15$ m; $z_1 = 0$; $z_2 = 10$ m; $p_1 = 260$ kPa, $V_1 = 3$ m/s; $\rho = 999$ kg/m^3.

From continuity equation, $A_1 V_1 = A_2 V_2$,

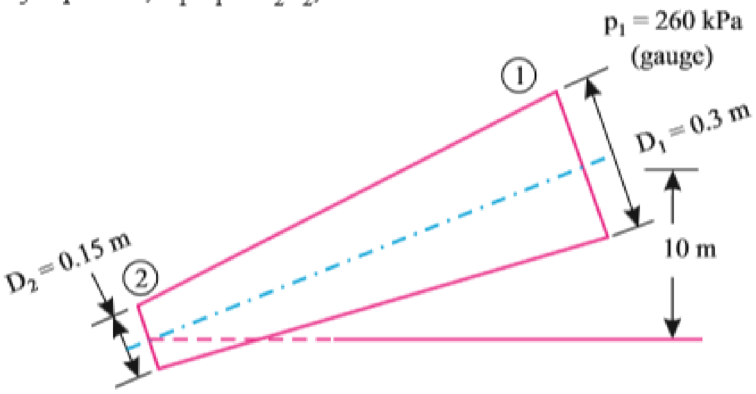

Fig. 6.7

$$V_2 = \frac{A_1}{A_2} \times V_1 = \left(\frac{\frac{\pi}{4} D_1^2}{\frac{\pi}{4} D_2^2} \right) \times V_1$$

$$= \left(\frac{D_1}{D_2} \right)^2 \times V_1 = \left(\frac{0.3}{0.15} \right)^2 \times 3 = 12 \text{ m/s}$$

Weight density of water, $w = \rho g = 999 \times 9.81 = 9800.19$ N/m^3

From Bernoulli's equation between sections 1 and 2 (neglecting friction effects as given), we have:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{260 \times 1000}{9800.19} + \frac{(3)^2}{2 \times 9.81} + 10$$

$$= \frac{p_2}{9800.19} + \frac{(12)^2}{2 \times 9.81} + 0$$

$$26.53 + 0.459 + 10 = \frac{p_2}{9800.19} + 7.34$$

or, $p_2 = 290566$ N/m^2 $=$ **290.56 kPa (Ans.)**

4. The water is flowing through a tapering pipe having diameters 300 mm and 150 mm at a sections 1 and 2 respectively. The discharge through the pipe is 40 litres/sec. The section 1 is 10 m above datum and section 2 is 6 m above datum. Find the intensity of pressure at section 2 if that at section 1 is 400 kN/m^2.

Example 6.7. *The water is flowing through a tapering pipe having diameters 300 mm and 150 mm at sections 1 and 2 respectively. The discharge through the pipe is 40 litres/sec. The section 1 is 10 m above datum and section 2 is 6 m above datum. Find the intensity of pressure at section 2 if that at section 1 is 400 kN/m^2.*

Solution. At Section 1:

$$\text{Diameter, } D_1 = 300 \text{ mm} = 0.3 \text{ m}$$

$$\therefore \quad \text{Area, } A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$$

$$\text{Pressure, } p_1 = 400 \text{ kN/m}^2$$

Height of upper end above the datum, $z_1 = 10$ m

At Section 2:

$$\text{Diameter, } D_2 = 150 \text{ mm} = 0.15 \text{ m}$$

$$\therefore \quad \text{Area, } A_2 = \frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$$

Height of lower end above the datum, $z_2 = 6$ m

Rate of flow (*i.e.*, discharge),

$$Q = 40 \text{ litres/sec} = \frac{40 \times 10^3}{10^6}$$

$$= 0.04 \text{ m}^3\text{/s}$$

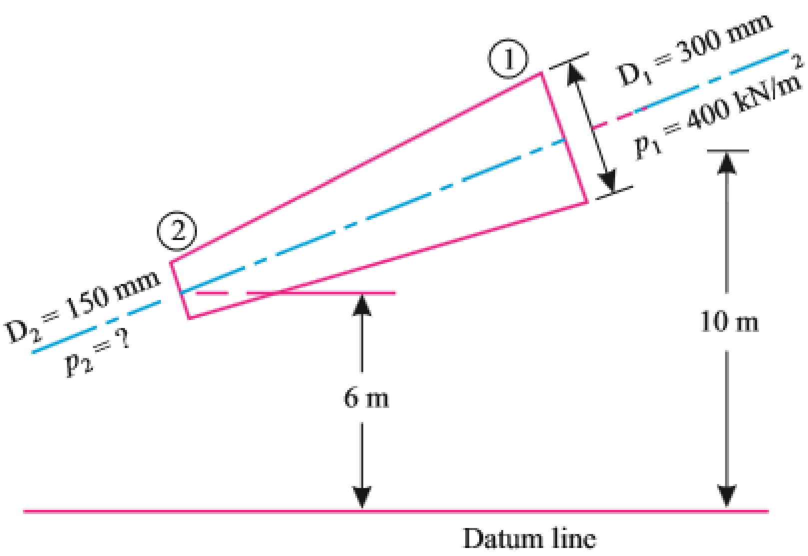

Fig. 6.8

Intensity of pressure at section 2, p_2:

Now, $$Q = A_1 V_1 = A_2 V_2$$

and, $$V_2 = \frac{Q}{A_2} = \frac{0.04}{0.01767} = 2.264 \text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

and, $$\frac{p_2}{w} = \frac{p_1}{w} + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) + (z_1 - z_2)$$

$$= \frac{400}{9.81} + \frac{1}{2 \times 9.81}(0.566^2 - 2.264^2) + (10 - 6)$$

$$(\because w = 9.81 \text{ kN/m}^3)$$

$$= 40.77 - 0.245 + 4 = 44.525 \text{ m}$$

$\therefore$ $$p_2 = 44.525 \times w = 44.525 \times 9.81 = \mathbf{436.8 \text{ kN/m}^2} \textbf{ (Ans.)}$$

5. A pipe 200 m long slopes down at 1 in 100 and tapers from 600 mm diameter at the higher end to 300 mm diameter at the lower end and carries 100 litres/sec of oil (sp. Gravity 0.8). if the pressure gauge at the higher end reads 60 kN/m^2. Neglect all losses.

   Determine (i) Velocities at the two ends

   (ii) pressure at the lower end.

Example 6.8. *A pipe 200 m long slopes down at 1 in 100 and tapers from 600 mm diameter at the higher end to 300 mm diameter at the lower end, and carries 100 litres/sec of oil (sp. gravity 0.8). If the pressure gauge at the higher end reads 60 kN/m², determine:*

(i) Velocities at the two ends;

(ii) Pressure at the lower end.

Neglect all losses.

Solution. Length of the pipe, $l = 200$ m; diameter of the pipe at the higher end, $D_1 = 600$ mm $= 0.6$ m,

$$\therefore \text{Area, } A_1 = \frac{\pi}{4} \times 0.6^2 = 0.283 \text{ m}^2$$

Diameter of the pipe at the lower end,

$$D_2 = 300 \text{ mm} = 0.3 \text{ m}$$

$$\therefore \text{Area, } A_2 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$$

Height of the higher end, above datum,

$$z_1 = \frac{1}{100} \times 200 = 2 \text{ m}$$

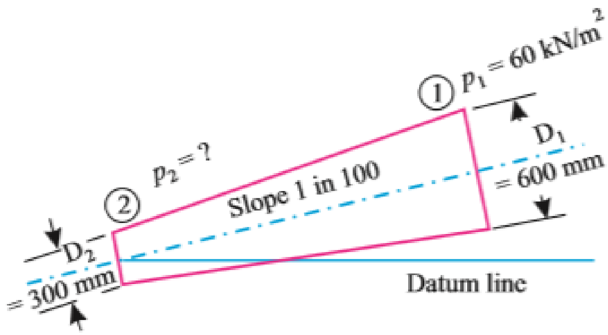

Fig. 6.9

Height of the lower end, above datum $z_2 = 0$

Rate of oil flow, $Q = 100$ litres/sec $= 0.1$ m³/s

Pressure at the higher end, $p_1 = 60$ kN/m²

(i) Velocities, V_1, V_2:

Now, $$Q = A_1 V_1 = A_2 V_2$$

where, V_1 and V_2 are the velocities at the higher and lower ends respectively.

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.1}{0.283} = \mathbf{0.353 \text{ m/s}} \text{ (Ans.)}$$

$$\text{and} \quad V_2 = \frac{Q}{A_2} = \frac{0.1}{0.0707} = \mathbf{1.414 \text{ m/s}} \text{ (Ans.)}$$

(ii) Pressure at the lower end p_2:

Using Bernoulli's equation for both ends of pipe, we have:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{60}{0.8 \times 9.81} + \frac{0.353^2}{2 \times 9.81} + 2 = \frac{p_2}{0.8 \times 9.81} + \frac{1.414^2}{2 \times 9.81} + 0$$

$$7.64 + 0.00635 + 2 = \frac{p_2}{0.8 \times 9.81} + 0.102$$

$$\therefore \quad \frac{p_2}{0.8 \times 9.81} = 9.54 \text{ m}$$

$$\text{or,} \quad p_2 = \mathbf{74.8 \text{ kN/m}^2} \text{ (Ans.)}$$

Chapter V (Flow through Pipes)

1. Calculate the discharge through a pipe of diameter 200 mm when the difference of pressure head between the two ends of a pipe 500 m apart is 4 m of water. Take the value of f=0.009 in in the formula h_f = $(4.f.L.V^2)/dx2g$

...lem 11.5 *Calculate the discharge through a pipe of diameter 200 mm when the difference of ...ure head between the two ends of a pipe 500 m apart is 4 m of water. Take the value of 0.009 in the formula* $h_f = \frac{4.f.L.V^2}{d \times 2g}$.

...olution. Given :

...ia. of pipe, $d = 200$ mm $= 0.20$ m

...ength of pipe, $L = 500$ m

...ifference of pressure head, $h_f = 4$ m of water

$f = .009$

...sing equation (11.1), we have $h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$

$$4.0 = \frac{4 \times .009 \times 500 \times V^2}{0.2 \times 2 \times 9.81} \text{ or } V^2 = \frac{4.0 \times 0.2 \times 2 \times 9.81}{4.0 \times .009 \times 500} = 0.872$$

$$V = \sqrt{0.872} = 0.9338 \simeq 0.934 \text{ m/s}$$

Discharge, Q = velocity × area

$$= 0.934 \times \frac{\pi}{4} d^2 = 0.934 \times \frac{\pi}{4} (0.2)^2$$

$= 0.0293$ m^3/s = **29.3 litres/s. Ans.**

2. An oil of specific gravity 0.9 and viscosity 0.06 poise is flowing through a pipe of diameter 200 mm at the rate of 60 litre / sec. Find the head lost due to friction for a 500 m length of pipe. Find the power required to maintain this flow.

Solution. Given :

Sp. gr. of oil $= 0.9$

Viscosity, $\mu = 0.06$ poise $= \frac{0.06}{10}$ Ns/m^2

Dia. of pipe, $d = 200$ mm $= 0.2$ m

Discharge, $Q = 60$ litres/s $= 0.06$ m^3/s

Length, $L = 500$ m

Density $\rho = 0.9 \times 1000 = 900$ kg/m^3

∴ Reynolds number, $R_e = \frac{\rho V d}{\mu} = 900 \times \frac{V \times 0.2}{\frac{0.06}{10}}$

where $V = \frac{Q}{\text{Area}} = \frac{0.06}{\frac{\pi}{4} d^2} = \frac{0.06}{\frac{\pi}{4}(.2)^2} = 1.909$ m/s $\simeq 1.91$ m/s

$$\therefore \quad R_e = 900 \times \frac{1.91 \times 0.2 \times 10}{0.06} = 57300$$

As R_e lies between 4000 and 10^5, the value of co-efficient of friction, f is given by

$$f = \frac{0.079}{R_e^{0.25}} = \frac{0.079}{(57300)^{0.25}} = .0051$$

$\therefore$ Head lost due to friction, $h_f = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times .0051 \times 500 \times 1.91^2}{0.2 \times 2 \times 9.81}$

$= \mathbf{9.48}$ **m of water. Ans.**

$\therefore$ Power required $= \frac{\rho g \cdot Q \cdot h_f}{1000} = \frac{900 \times 9.81 \times 0.06 \times 9.48}{1000} = \mathbf{5.02}$ **kW. Ans.**

3. Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 litres/sec.

Solution. Given :

Dia. of smaller pipe, $D_1 = 200 \text{ mm} = 0.20 \text{ m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.03141 \text{ m}^2$

Dia. of large pipe, $D_2 = 400 \text{ mm} = 0.4 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564 \text{ m}^2$

Discharge, $Q = 250 \text{ litres/s} = 0.25 \text{ m}^3/\text{s}$

Velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96 \text{ m/s}$

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{.12564} = 1.99 \text{ m/s}$

Loss of head due to enlargement is given by equation (11.5) as

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g} = \mathbf{1.816 \text{ m of water.}}$$

4. The rate of flow of water through a horizontal pipe is 0.25 m^3/s. The diameter of the pipe which is 200 mm is suddenly enlarged to 400 mm. Determine loss of head due to sudden enlargement (h_e)

Solution. Given :

Dia. of smaller pipe, $D_1 = 200 \text{ mm} = 0.20 \text{ m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.03141 \text{ m}^2$

Dia. of large pipe, $D_2 = 400 \text{ mm} = 0.4 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564 \text{ m}^2$

Discharge, $Q = 250 \text{ litres/s} = 0.25 \text{ m}^3/\text{s}$

Velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96 \text{ m/s}$

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{.12564} = 1.99 \text{ m/s}$

Loss of head due to enlargement is given by equation (11.5) as

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g} = \textbf{1.816 m of water.}$$

5. A 150 mm diameter pipe reduces in diameter abruptly to 100 mm diameter. If the pipe carries water at 30 litres per second, calculate the pressure loss across the contraction. Take the co-efficient of contraction as 0.6

Solution. Given :

Dia. of large pipe, $D_1 = 150 \text{ mm} = 0.15 \text{ m}$

Area of large pipe, $A_1 = \frac{\pi}{4} (.15)^2 = 0.01767 \text{ m}^2$

Dia. of smaller pipe, $D_2 = 100 \text{ mm} = 1.10 \text{ m}$

Area of smaller pipe, $A_2 = \frac{\pi}{4} (.10)^2 = 0.007854 \text{ m}^2$

Discharge, $Q = 30 \text{ litres/s} = .03 \text{ m}^3/\text{s}$

Co-efficient of contraction, $C_c = 0.6$

From continuity equation, we have $A_1V_1 = A_2V_2 = Q$

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.03}{.01767} = 1.697 \text{ m/s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{.03}{.007854} = 3.82 \text{ m/s}$$

Applying Bernoulli's equation before and after contraction,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c \qquad ...(i)$$

But $Z_1 = Z_2$

and h_c, the head loss due to contraction is given by equation (11.6) as

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = \frac{3.82^2}{2 \times 9.81}\left[\frac{1}{0.6} - 1\right]^2 = 0.33$$

Substituting these values in equation (*i*), we get

$$\frac{p_1}{\rho g} + \frac{1.697^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{3.82^2}{2 \times 9.81} + 0.33$$

or

$$\frac{p_1}{\rho g} + 0.1467 = \frac{p_2}{\rho g} + .7438 + .33$$

$$\therefore \quad \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = .7438 + .33 - .1467 = 0.9271 \text{ m of water}$$

$$\therefore \quad (p_1 - p_2) = \rho g \times 0.9271 = 1000 \times 9.81 \times 0.9271 \text{ N/m}^2$$

$$= 0.909 \times 10^4 \text{ N/m}^2 = 0.909 \text{ N/cm}^2$$

$\therefore$ Pressure loss across contraction $= p_1 - p_2 = \textbf{0.909 N/cm}^2$. **Ans.**